low liquid flow rates, indicating an increased film stability due to viscous effects. This trend has not yet been analysed quantitatively with any certainty. The curves can be approximated over much of their length by straight lines and one can arbitrarily define the "critical gas velocity" as the point at which these lines hit the axis.

Figure 3 shows some of Steen's data with an air–water system at several pressures. In this case the representations in terms of π_1 or π_2 coincide. The trend with pressure is consistent with both theories but the data are displaced somewhat from the correlation (a lot less than the maximum scatter in the original graph, however).

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REFERENCES

1. I. I. PALEEV and B. S. FILIPPOVICH, Phenomena of liquid transfer in two-phase dispersed annular flow, *Int. J. Heat Mass Transfer* **9**, 1089–1093 (1966).
2. G. B. WALLIS, The onset of droplet entrainment in annular gas–liquid flow, Report No. 62GL127, General Electric Co., Schenectady, N.Y. (1962).
3. D. A. STEEN and G. B. WALLIS, The transition from annular to annular-mist co-current two-phase downflow, U.S.-Euratom Report EURAEC-1147 (June 1964).
4. D. A. STEEN, M.S. Thesis, Dartmouth College (1964).
5. M. WICKS and A. E. DUKLER, Entrainment and pressure drop in co-current gas–liquid flow, *A.I.Ch.E. Jl* **6**, 463 (1960).

Int. J. Heat Mass Transfer. Vol. 11, pp. 785–786. Pergamon Press 1968. Printed in Great Britain

PHENOMENA OF LIQUID TRANSFER IN TWO-PHASE DISPERSED ANNULAR FLOW
COMMENTS ON G. B. WALLIS' DISCUSSION

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IN HIS discussion Wallis' points concerning the paper of Paleev and Filippovich are well taken. His comparison of viscosity effects in Fig. 2 clearly shows the inapplicability of the Paleev and Filippovich expression π_1 to correlate the quantity of entrainment. Wallis claims that both π_1 and π_2 show the correct trends with pressure, although in Fig. 3 the percent entrainment is plotted versus air velocity, V_G, only. His point would have been more strongly made by using $V_G\sqrt{(\rho_G)}$ as the abscissa instead of V_G.

Both groups π_1 and π_2 suffer from the absence of any dependence on liquid rate, although this is a variable of major importance, as Wallis' Fig. 2 shows.

A more general dimensional analysis consideration might result from inclusion of other relevant forces, also. These might include viscous forces in the gas and inertial forces in the liquid. The following table shows the formulas for each of these, normalized by the interfacial forces:

		Forces: inertial	Forces: viscous
Phase	Gas	$R_1 = \dfrac{\rho_G V_G^2 L}{\sigma g_c}$	$R_3 = \dfrac{\mu_G^2}{\rho_G \sigma g_c L}$
	liquid	$R_4 = \dfrac{\rho_L V_L^2 L}{\sigma g_c}$	$R_2 = \dfrac{\mu_L^2}{\rho_L \sigma g_c L}$

Paleev and Filippovich argue that π_1 should be a correlating group because in terms of the above groups, (1) R_1 and R_2 are the "determining criteria" for gas–liquid interactions in spray-nozzle applications, and (2) the product $R_1 R_2$ does not involve the characteristic length L, which must vanish for a liquid of infinite depth.

As substitution shows,

$$\pi_1 = R_1 R_2$$

Wallis' group π_2 can be expressed as

$$\pi_2 = \frac{R_1^2 R_3}{R_4}$$

assuming liquid and gas velocities are equal, which may not be a bad assumption for the droplets already in the gas phase; for the droplets being generated from the liquid film surface, however, equal gas and liquid velocity is not the case. The effect of liquid flow rate thus could enter via the term R_4.

Both π_1 and π_2 do not involve L, the characteristic length dimension. Indeed, it is possible to show that there is an infinite set of powers a_i ($i = 1, 2, 3, 4$) to which the R_i's can be raised which lead to a product R

$$R = R_1^{a_1} R_2^{a_2} R_3^{a_3} R_4^{a_4}$$

which is independent of the length L since this requires only that

$$a_1 - a_2 - a_3 + a_4 = 0.$$

Moreover, this does not seem to be a valid objective, anyway, since two-phase films characteristic of the entrained regime are typically of the order of 5–50 thousandths of an inch thick—hardly infinite in comparison with the length scales of the droplets torn from the film or the wave heights at the surface!

It seems then, that ordinary dimensional analysis without any constraints on the relevance of the various forces or information on the proper length scales is sterile in producing useful general correlations of entrainment. What is needed is either an approach based on the concepts of *similarity** (as distinguished from dimensional analysis) or a more fundamental attack seeking to identify and describe the action of the forces which generate the drops and maintain them in the gas phase.

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* As described, e.g. in the book by S. J. KLINE, Similitude and approximation theory. McGraw-Hill, New York (1965).